

## Calculating the Annual Return (Realized Compound Yield) on a Coupon Bond

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Objective:

To show that the annual return actually earned when holding a coupon-bearing bond to maturity will equal its yield to maturity only if you can and do reinvest the coupons at the yield to maturity.

### PROOF FOR ANNUAL PAY BONDS

1. Assume:  $F = 1000$     $C = \$80$     $t=4$  years
2. If  $P=100$  we know that  $YTM = 8\%$
3. Definition of Annual Return

$$r_{ann} = \left( \frac{V_t}{V_0} \right)^{1/t} - 1,$$

where  $V_t = \$$  amount at the end and  $V_0$  is the  $\$$  amount at the beginning. In our case

$$V_0 = \$1000 \quad \text{and} \quad t = 4$$

therefore

$$r_{ann} = \left( \frac{V_t}{\$1000} \right)^{1/4} - 1$$

4. To calculate  $r_{ann}$  we must calculate  $V_t$ . To calculate  $V_t$  we must account for the reinvestment of the annual 8% coupon (= \$80 per annum). Assuming we reinvest these coupons at 8%, we have the following cash flows on the bond:

#### Cash Flows

	Yr 1	Yr 2	Yr 3	Reinvest	Yr 4
1st coupon	\$80			* $(1.08)^3$	= \$100.78
2nd coupon		\$80		* $(1.08)^2$	= \$93.31
3rd coupon			\$80	* $(1.08)$	= \$86.40
4th coupon + principal					\$1080.00
FINAL TOTAL ( $V_t$ ) =					\$1360.49

5. In this case,

$$r_{ann} = \left( \frac{\$1360.49}{\$1000} \right)^{1/4} - 1 = .08$$

Thus

$r_{ann} = YTM$  if you reinvest the coupons at the YTM

6. If you reinvest the coupons at more than 8% you accumulate more than \$1360.49 and earn an annual return  $> .08$  and if you reinvest the coupon at less than 8% you accumulate less than \$1360.49 and earn an annual return  $< .08$ .

#### FOR SEMI-ANNUAL PAY BONDS: AN EXERCISE

1. Assume:  $F = \$1000$                        $C/2 = \$40$                        $t = 4$  years
2. If  $P = 100$  we know  $YTM = 8\%$
3. Calculate the annual return assuming you reinvest the coupons at the  $YTM/2$  or at  $.08/2 = .04$
4. What is the relationship between YTM and  $r_{ann}$  in this case?