

## Gains from Diversification: A Two-Security Illustration

W. L. Silber

The nice thing about diversification is that it almost always produces gains to a portfolio in the form of increased return that exceeds the cost in terms of increased standard deviation. The only exception is the case of correlation equal to one. This is nicely illustrated with a simple numerical example.

Recall that the expected return,  $R$ , on a two-asset portfolio is:

$$(1) \quad R = X_1R_1 + X_2R_2 ,$$

where  $R_1$  and  $R_2$  are the expected returns on security 1 and 2 and  $X_1$  and  $X_2$  are the weights invested in each.

The standard deviation ( $s$ ) on the portfolio is:

$$(2) \quad s = [X_1^2s_1^2 + X_2^2s_2^2 + 2X_1X_2s_1s_2r]^{1/2} ,$$

where  $s_1$  and  $s_2$  are the standard deviations of asset 1 and 2 and  $r$  is the correlation of returns.

Suppose we have the following information:

$$R_1 = .13 \quad s_1 = .064$$

$$R_2 = .24 \quad s_2 = .136$$

If  $X_1 = 1$  and  $X_2 = 0$ , then the return on the portfolio is .13 and the standard deviation is .064, all the same as asset 1.

Now consider a portfolio that has  $X_1 = .5$  and  $X_2 = .5$ , so that we add the riskier security 2 until it makes up half the portfolio.

We know from equation (1) that the new return  $R$  is half way between  $R_1$  and  $R_2$

$$R = .5(.13) + .5(.24) = .185$$

The standard deviation of the half-and-half portfolio, however, will always be less than half way between  $s_1$  and  $s_2$  as long as  $r < 1$ . When  $r = 1$ , the standard deviation of the half-and-half portfolio is exactly half way between the two.

These relationships are illustrated by entering the values for  $X_1$ ,  $X_2$ ,  $s_1$ , and  $s_2$  from the above into equation (2) combined with alternative values for  $r$ . In particular, we have the following results:

$r$	$s$
.2	.0807
.5	.0885
.8	.0955
.99	.0998
1.0	.1

Thus, as long as  $r < 1$ , the  $s$  of the combined portfolio is less than .1 (which is midway between  $\sigma_1 = .064$  and  $\sigma_2 = .136$ ). Thus as long as  $r < 1$  there are gains from diversification.

Note also that diversification can never create a “monster portfolio.” That is, the standard deviation never “explodes” because it never exceeds a weighted average of the two underlying standard deviations. The reason is that  $r$  can never be greater than one. Thus the largest possible standard deviation is a weighted average of  $s_1$  and  $s_2$ .