FROM VAN HORNE, FINANCIAL MARKETS RATES + FLOWS, 4TH EDITION, PRENTICE HALL.

Forward Rates of Interest

The rate of interest, or yield, on an actual bond of a particular maturity is known as its *spot rate of interest*. (See Chapter 3 for the mathematics of yields). When considering the expectations theory, it is useful to transform spot rates of interest into *forward rates*. In a formal contract, the forward rate is that rate at which two parties agree to lend and borrow money for a specified period of time in the future. For example, a *forward contract* might be for a two-year loan beginning one year in the future, or for a one-year loan beginning three years hence. Such an arrangement represents an explicit contract.

Forward rates also can be implicit. Implied in the term structure at any moment is a set of forward rates.[2]

$$(1+{}_tR_n)^n = (1+{}_tR_1)(1+{}_{t+1}r_{1t})(1+{}_{t+2}r_{1t})\cdots(1+{}_{t+n-1}r_{1t}) \tag{5–1}$$

where ${}_tR_n$ represents the actual spot rate of interest at time t on an n-period loan, ${}_tR_1$ is the actual rate on a one-period loan at time t, and ${}_{t+1}r_{1t}$, ${}_{t+2}r_{1t}$, and ${}_{t+n-1}r_{1t}$ are forward rates for one-period loans beginning at times $t+1$, $t+2$, and $t+n-1$, implied in the term structure at time t. Thus, a loan for four years is equivalent to a one-year loan plus a series of forward contracts, each renewing the loan for a successive year. The formula for deriving the one-period forward rate beginning at time $t+n$, implied in the term structure at time t, is

$$\begin{aligned} 1+{}_{t+n}r_{1t} &= \frac{(1+{}_tR_{1t})(1+{}_{t+1}r_{1t})\cdots(1+{}_{t+n-1}r_{1t})(1+{}_{t+n}r_{1t})}{(1+{}_tR_{1t})(1+{}_{t+1}r_{1t})\cdots(1+{}_{t+n-1}r_{1t})} \\ &= \frac{(1+{}_tR_{n+1})^{n+1}}{(1+{}_tR_n)^n} \end{aligned} \tag{5–2}$$

$${}_{t+n}r_{1t} = \frac{(1+{}_tR_{n+1})^{n+1}}{(1+{}_tR_n)^n} - 1$$

[1]Irving Fisher, "Appreciation and Interest," *Publications of the American Economic Association*, XI (August 1986), 23–29, 91–92; and F. A. Lutz, "The Structure of Interest Rates," *Quarterly Journal of Economics*, LV (November 1940), 36–63.

[2]For such a derivation, see, for example, J. R. Hicks, *Value and Capital*, 2nd ed. (London: Oxford University Press, 1946), pp. 141–45. If coupon bonds were involved, the formula implicitly assumes that the coupon payments are reinvested, the lender receiving the principal and reinvesting interest at maturity. The formula contrasts with one in which interest and principal payments are discounted back to present value in accordance with the times when they are to be received.