IMMUNIZATION:
A NUMERICAL EXAMPLE

FROM
J.C. Van Horne
Financial Market Rates and Flows
Prentice Hall, 1998

166

Immunization with Coupon Issues

Whenever possible, look first to pure discount Treasury securities. A large number of them presently exist. However, because of gaps in maturity or because of liquidity or pricing difficulties, you may have to turn to coupon bonds. Here we can also immunize, though usually less than perfectly.

With coupon bonds, two types of risk are germane: *price risk* and *coupon reinvestment risk*. Price risk is the risk that—with changing interest rates—the bond will need to be sold at a different price from what was expected. Of course, price risk could be reduced to zero if bonds whose maturity equaled the intended holding period were selected. However, there would still be coupon reinvestment risk, which is the risk associated with reinvesting the coupons received at yields that are different from the yield of the bond when it was purchased.

Together, these two risks represent the total risk associated with a bond investment for an investor with an intended holding period. Moreover, these risks work in opposite directions. An increase in interest rates reduces the price of a bond but increases the yield possible from reinvestment of coupons. In contrast, a decline in interest rates results in a price increase but lowers the yield possible from coupon reinvestment. Thus, the two types of risk are offsetting. To "immunize" a bond investment from subsequent interest-rate changes, these two risks must be balanced so that they are completely offsetting.

An Illustration

To envision this notion, suppose that $1,210 must be realized in two years.[7] At present, the term structure is flat with a 10 percent yield-to-maturity throughout. However, at the end of the first year, there is a possibility of a one-time parallel shift in interest rates, either to 12 percent or to 8 percent, as well as the possibility that interest rates will remain unchanged. Available for investment is a two-year bond with a 10 percent annual coupon. For an investment in this bond, the amounts realized at the end of two years under the three possible states of the world are as follows:

	Term Structure Level		
	8%	*10%*	*12%*
Principal payment	$1,000	$1,000	$1,000
Reinvested first-year coupon	108	110	112
Second-year coupon	100	100	100
	$1,208	$1,210	$1,212

[6]Frederick M. Redington, "Review of the Principle of Life Office Valuations," *Journal of the Institute of Actuaries*, 18 (1952), 286–340, was the first to develop the concept and to use the word *immunization*.

[7]This example comes from Jeffrey Skelton, "Recent Results in Term Structure Theory" (Berkeley Program in Finance Seminar, September 13, 1982).

Only if interest rates remain at 10 percent can the $1,210 desired at the end of the holding period be realized. If there is a shift in interest rates, the bond will not provide perfect immunization. The reason is that the reinvestment rate on the first-year coupon payment changes.

Suppose, however, that there is also a 10 percent coupon bond with 2.1 years to maturity. Assume that this bond pays a $10 coupon at maturity (one tenth of a full-year coupon), together with $100 at the end of the first and second years. In this case, the bond will need to be sold before maturity. If interest rates turn out to be 8 percent, the market value of the bond at the end of two years, with one tenth of a year to go, will be approximately $1,002. That is, the $1,000 principal amount and $10 coupon at the end of one tenth of a year will be worth approximately $1,002 at the beginning of the period with an 8 percent discount rate. Contrarily, the market value of the bond at the end of two years will be approximately $998 if the term structure shifts from 10 percent to 12 percent. Therefore, the amounts realized at the end of two years for a 2.1-year bond will be

	Term Structure Level		
	8%	*10%*	*12%*
Market value on sale	$1,002	$1,000	$998
Reinvested first-year coupon	108	110	112
Second-year coupon	100	100	100
	$1,210	$1,210	$1,210

In this case, the bond is perfectly immunized in the sense that the amounts received are exactly the same under all possible states. The changes in principal value at the end of two years exactly offset the changes in reinvestment rate on the first-year coupon.

Using Eq. (7-1), the duration of the 2.1-year bond is

$$D = \frac{\dfrac{\$100(1)}{(1.10)} + \dfrac{\$100(2)}{(1.10)^2} + \dfrac{\$1{,}010(2.1)}{(1.10)^{2.1}}}{\dfrac{\$100}{(1.10)} + \dfrac{100}{(1.10)^2} + \dfrac{\$1{,}010}{(1.10)^{2.1}}} = 2.0 \text{ years}$$

Thus, the duration is equal to the intended holding period. Whenever the term structure is flat and only parallel shifts in interest rates occur, a bond investment is always immunized when its duration equals the intended holding period.[8] Moreover, one would want to immunize with noncallable bonds. For callable bonds, the actual horizon often is less than maturity due to the bond being called (see chapter 12).

Finally, as interest rates shift, so too does the duration of the portfolio, as illustrated in the previous section. To restore the desired duration, we must reimmunize the portfolio by adding and subtracting bonds. However, there are transactions costs to doing so. Consequently, there are trade-offs between how much slippage in duration is tolerable before reimmunization is necessary.

[8]For proof of this statement, see G. O. Bierwag and George G. Kaufman, "Coping with the Risk of Interest-Rate Fluctuations: A Note," *Journal of Business*, 50 (July 1977), 364–70; and Paul H. Samuelson, "The Effect of Interest-Rate Increases on the Banking System," *American Economic Review*, 35 (March 1945), Appendix B.