**Approximating the value of an at-the-money forward option: An application of the intuition that an ATM call option is approximately equal to the expected value of the positive payoffs at the end**

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There may come a time when you will have to value an option in real life, in a situation where you may not have an excel spreadsheet with the Black-Scholes formula at your disposal. Perhaps you are bargaining over a rental rate, and the owner offers to throw in an option to purchase the property in one year at the current market price. Or perhaps you are negotiating your salary, and want to get a rough idea of what the options in the package are worth.

We demonstrate that you can approximate the value of a call option by using the intuition that a call is worth the expected value of the positive payoffs at the expiration date of the option. We assume in this example that the call option has one year to expiration with a strike or exercise price (E) of 100. Also, we assume the underlying stock has a forward price (S) of 100, so that we are valuing an at-the-money option, and can therefore ignore the interest rate component of the option (this would also work if the current price of the asset is 100 and the one year interest rate is very low). Finally, we assume the annual volatility (standard deviation) of the underlying asset is 20%.

We begin by thinking about the distribution of potential stock prices at the option expiration date. We will then estimate the call value by estimating the call option's payoffs for the different possibilities of the expiration stock price, weighted by the probabilities of the payoffs as determined by the distribution.

**Distribution of Potential Stock Prices:**

We can simplify the approach by assuming that stock prices follow a normal distribution. From statistics, a normal distribution is defined by its mean and its standard deviation. Also recall that with a normal distribution, roughly 68% of outcomes fall within one standard deviation of the mean, 95% fall within two standard deviations, and 99% fall within three standard deviations.

In one year, when the call option expires, the mean of the distribution of potential stock prices is the current forward price of the stock, and the standard deviation is the volatility assumed for the time until expiration.* With a forward price of $100, and a one year standard deviation of 20%, we can imagine the potential future stock price to be distributed as follows in one year:

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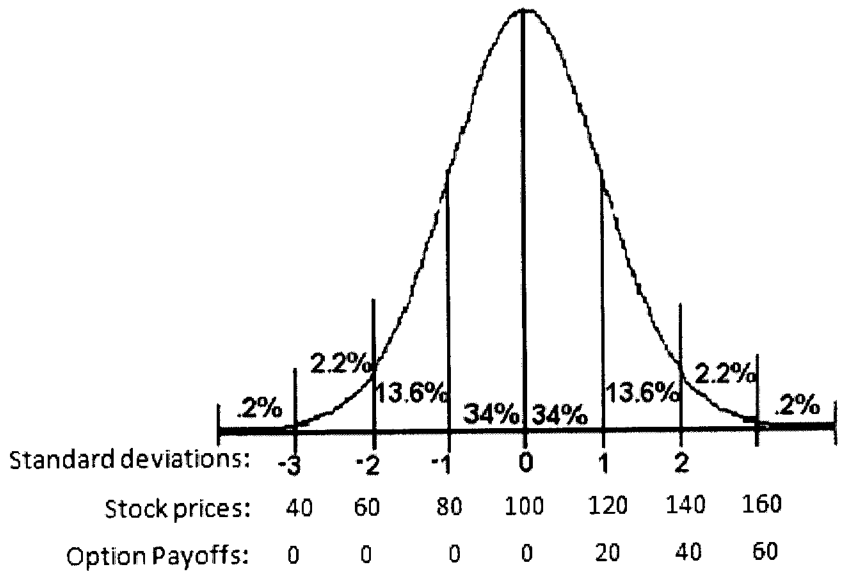
* We assumed that the annual volatility of the underlying stock is 20%. When you are calculating option values on the fly, you may have to adjust this value depending on what kind of asset your option is on. See the historical one-year volatilities in the Appendix for various sectors to get a rough idea of what volatilities have been like in the past. For options longer than a year, remember that volatilities scale proportionately with the square root of time. For instance, a two year option's volatility would be (one year volatility) x (sqrt(2)).

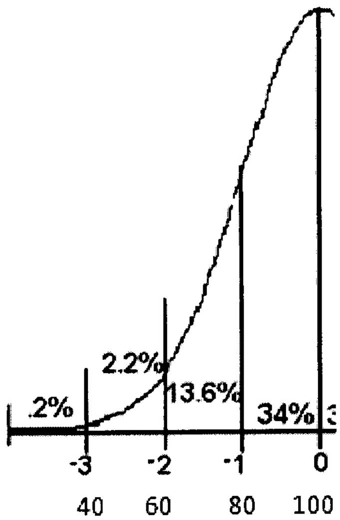



**Calculating the Option Value**

The fundamentals behind the estimation procedure are rooted in what you already know about options. By having the right but not the obligation to buy the stock for the strike price of $100, you will not lose anything if the stock winds up below 100, but will gain (S – E) dollars if the stock price finishes above $100.

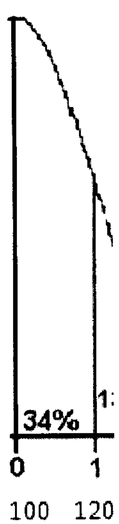
Using all the information we have so far, we can approximate the call option value by breaking the distribution of potential outcomes into regions. We can then determine what the payoff to the call would be if the stock price ended up in the region, and weight it by the probability of the stock price ending up in that region.

*Any outcome left of the $100 strike price:*




We can ignore these outcomes because our call payoff will be zero, and thus their occurrence will not add any value to the call (and, of course, will not decrease the value of the call).

*The region between the forward price and one standard deviation above the forward:*




In words, we can say that there is a 34% chance that the stock will have a value between 100 and 120 at the expiration of the call option. Therefore there is a 34% chance that the call will pay off between $0 and $20. We use the formula:

Value added to Call = Likelihood of payoff x Expected Value of Payoff

We can estimate the expected value of these outcomes as .34 x (20 + 0)/2 = $3.4

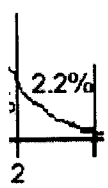
*The region between one standard deviation and two standard deviations above the forward price:*


120 140

We can repeat the previous procedure for stock prices between one and two standard deviations above the expected forward price:

We calculate the expected value of these outcomes as .136 x (40 + 20)/2 = $4.08

*The region between two standard deviations and three standard deviations above the forward price:*




140 160

The expected value of these outcomes equals .022 x (40+60)/2 = $1.1

We ignore outcomes above three standard deviations as the contribution to the call value is small.

We now can sum the values of the outcomes to obtain our estimate of the value of the call option.
C = $3.4 + $4.08 + $1.1
C = $8.58

A Comparison with Black-Scholes: A formal calculation using a Black-Scholes calculator with S=100, E=100, r=0, t=1, and a standard deviation of 20 %, gives a value of $7.97. Conclusion: our approximation puts us in the ballpark.

**Appendix –**

Historical equity price volatilities for various industries on an annual basis:[†]

| | S&P500 | MSCI World | Technology | Utilities | Health Care | Retail | Financials |
|---|---|---|---|---|---|---|---|
| 12/31/1990 | 15.90% | 15.83% | | | | 23.75% | 22.64% |
| 12/31/1991 | 14.25% | 13.28% | | 10.52% | | 22.07% | 19.19% |
| 12/31/1992 | 9.70% | 10.41% | | 8.03% | | 15.01% | 12.70% |
| 12/31/1993 | 8.62% | 8.74% | | 9.79% | | 17.32% | 14.33% |
| 12/31/1994 | 9.85% | 7.69% | | 14.52% | | 15.93% | 12.84% |
| 12/31/1995 | 7.80% | 7.84% | | 10.52% | | 14.10% | 12.66% |
| 12/31/1996 | 11.82% | 7.14% | | 11.46% | | 19.85% | 16.62% |
| 12/31/1997 | 18.20% | 12.41% | | 12.60% | | 20.45% | 22.75% |
| 12/31/1998 | 20.34% | 16.31% | | 14.78% | | 29.92% | 29.26% |
| 12/31/1999 | 18.05% | 11.93% | | 16.43% | | 27.98% | 26.77% |
| 12/31/2000 | 22.23% | 15.13% | | 24.88% | | 42.27% | 33.13% |
| 12/31/2001 | 21.46% | 16.67% | 54.77% | 23.39% | | 33.15% | 24.32% |
| 12/31/2002 | 25.96% | 20.62% | 44.80% | 31.62% | | 31.02% | 30.54% |
| 12/31/2003 | 17.04% | 13.32% | 27.46% | 16.06% | 16.65% | 21.72% | 19.30% |
| 12/31/2004 | 11.09% | 9.59% | 19.59% | 11.60% | 11.67% | 16.03% | 12.17% |
| 12/31/2005 | 10.28% | 7.82% | 12.98% | 14.58% | 9.40% | 16.60% | 11.46% |
| 12/31/2006 | 10.02% | 9.76% | 15.93% | 11.63% | 9.99% | 15.88% | 11.38% |
| 12/31/2007 | 16.00% | 12.89% | 18.04% | 18.19% | 11.96% | 21.35% | 23.51% |
| 12/31/2008 | 41.08% | 33.10% | 39.09% | 37.35% | 30.54% | 49.21% | 72.86% |
| 12/31/2009 | 27.27% | 22.94% | 26.62% | 20.70% | 18.79% | 32.24% | 66.87% |

[†] The data in this table come from daily observations on returns from the Bloomberg data base.