

Liquidity Premium Theory of the Term Structure

William Silber

- Given: (1) ${}_tR_1 = .02$
- (2) $\text{Exp } {}_{t+1}R_1 = \text{Mean of the following distribution:}$

${}_{t+1}R_1$	Prob
.05	.25
.04	.50
.03	.25

$$\text{Exp } {}_{t+1}R_1 = [.05(.25) + .04(.5) + .03(.25)] = .04$$

To Prove: If investors are risk-averse, then equilibrium ${}_tR_2 > [(1.02)(1.04)]^{1/2} - 1$. In words, we want to prove that the equilibrium ${}_tR_2$ is greater than the “pure expectations” answer that ${}_tR_2$ is .03, a geometric average of the current and expected future short-term rate.

This is equivalent to saying that the equilibrium price of ${}_tR_2$ is less than the pure expectations theory price:

$$P_2 = \frac{100}{(1.03)^2} = 94.2596$$

Approach: We start by assuming $P_2 = 94.2596$ and show that risk averse one year investors will sell ${}_tR_2$ if its price is 94.2596, forcing down the price and pushing up the yield. This creates a liquidity premium in ${}_tR_2$.

Focus: One-year investor decision-making

Strategy 1

Buy ${}_tR_1$

Results: $HPY = .02$ with certainty

Strategy 2

Buy ${}_tR_2$ and sell after one year.

The expected selling price after one year (given $\text{Exp } {}_{t+1}R_1 = .04$) is:

$$\frac{100}{1.04} = 96.154$$

The expected $HPY = \frac{96.154}{94.2596} = .02$ but it is uncertain.

Next year's price could be:

$$\frac{100}{1.05} = 95.24 \quad \text{or} \quad \frac{100}{1.03} = 97.09$$

Thus, the HPY could be $\frac{95.24}{94.2596} = .01$ or $\frac{97.09}{94.2596} = .03$

Risk averse one year investors prefer buying ${}_tR_1$ and earning 2% with certainty to buying ${}_tR_2$ and selling after one year, which earns them an "uncertain 2% expected HPY ."

Outcome:

One year investors sell ${}_tR_2$ if the price is 94.2596.

The selling pressure forces down the price and drives up the yield. Suppose the price drops to 93.35. One year investors now check their expected HPY of buying ${}_tR_2$ at 93.35 and selling after one year:

$$HPY = \frac{96.154}{93.35} = .03$$

If the extra 1% expected HPY (.03 versus .02) of this uncertain strategy compensates for their risk aversion then they stop selling ${}_tR_2$.

Conclusion:

New equilibrium $P_2 = 93.35$

New equilibrium ${}_tR_2$ is

$$\left(\frac{100}{93.35} \right)^{1/2} - 1 = .035$$

Therefore,

New equilibrium ${}_t R_2 > [(1.02)(1.04)]^{1/2} - 1$

Additional Notes:

1) New forward rate

$${}_{t+1}r_1 = \frac{(1.035)^2}{(1.02)} - 1 = .05$$

The forward rate is greater than the expected rate.

2) The forward rate will exceed the expected rate by a larger amount the greater the degree of risk aversion of one year investors (because the price of ${}_t R_2$ will be lower and its yield will be higher).