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A Note on Calculating Loss Probabilities from Means and Standard Deviations:

An Example from the S&P 500


1.      Modern portfolio theory identifies the risk and return on a portfolio with the mean
        and standard deviation of returns on the portfolio.  Most investors think about risk
        in *terms* of the likelihood of losing money.  This note shows that it is possible to
        transform means and standard deviations into loss probabilities.  The numerical
        examples use realistic numbers for investing in the stock market.


2.      Between 1926 and 2015, the S&P 500 had an approximate mean return, $\mu$ = .12,
        and a standard deviation, $\sigma$ =.20. We take these as our best estimates of the true
        parameters so we can use them to describe the future distribution of returns.
        Assuming these numbers come from a normal distribution, what is the probability
        of losing money in any one year when invested in the S&P 500?


3.      We know from statistics that $Z = \dfrac{X - \mu}{\sigma}$ is a standardized normal variable with
        mean zero and standard deviation of one.  What this means is that we can
        determine the probability of observing specific outcomes by substituting numbers
        *into* the formula for Z and looking up the probability in a normal distribution table,
        sometimes called a table of normal curve areas.  For example, in the table below,
        each entry gives the probability of observing an outcome between X and the
        mean.  Thus, if we take $\mu$ = .1 2 and $\sigma$ = .20 as the true parameters, we can
        determine the probability of observing a specific outcome between *X = .02*
        and .12 as follows:

$$Z = \frac{.02 - .12}{.20} = \frac{-.1}{.2} = -.5$$

The Z score indicates that an observed return of .02 (2 percent) is .5 standard deviations below the mean. Our table of normal curve areas has an entry of .1915 for Z = .5. Because the normal distribution is symmetric, the probability of observing a return between the mean and .5 standard deviations <u>below</u> the mean is also .1 915, or 19 percent.

4.   To calculate the probability of losing money in any one year, we proceed in the following steps: (a) Because normal distributions are symmetric, we know that .5 is the probability of observing an outcome below the mean; (b) We can calculate the probability of observing a return below the mean but above zero by substituting X = 0 in the Z statistic and looking up the probability of observing that value of Z in the accompanying normal distribution table; (c) We then subtract that probability from .5 to produce the probability of observing a return less than zero, i.e., the probability of losing money.

5.   In our case of the S&P 500, we have:

$$z = \frac{0 - .12}{.2} = -.6$$

From our Normal Distribution table:

$$\Pr ob\,(Z = -.6) = \Pr ob\,(Z = .6) = .2257$$

Thus, the probability of observing an outcome between .12 and zero is .2257. Therefore the probability of losing money in any one year from an investment in the S&P 500 is .5 - .2257 = .2743.

6.   From this it follows that the probability of making money in any year is .7257. This number comes from adding the probability of outcomes above the mean, .5, to the

probability of <u>positive</u> outcomes below the mean, .2257. Therefore, the odds of making money in any one year in the S&P 500 is $\frac{.7257}{.2743}$ or about 2.6: 1.

7.   Although the odds of making money are better than 2:1, the probability of losing money in any one year is more than 27%, which is substantial. There are two additional caveats when calculating the odds of losing money:

(a) If the true distribution of returns has fat tails, so that the probability of outcomes beyond a Z-score of, say, 1.65 is higher than in the normal distribution, then the probability of losing money is even greater.

(b) The probability of losing money is just half the story. How much you lose, the expected value of the loss, is often more important. A .05 chance of losing 5 percent of your investment is not nearly as bad as a .02 chance of losing 100 percent. Given the parameters for the S&P500 we can calculate the expected value of the loss (assuming the return is negative) as long as the distribution of returns is normal. It turns out to be -12.3%

TABLE 3   Normal Curve Areas
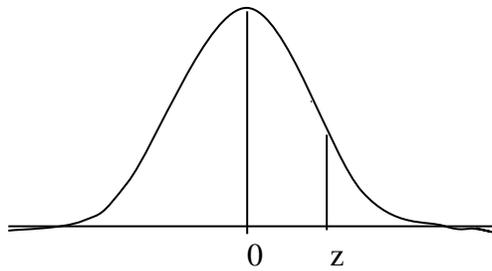


| | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3196 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3969 | .3898 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4136 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4419 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4921 | .4826 | .4930 | .4934 | .4839 | .4942 | .4946 | .4950 | .4854 | .4957 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4979 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |