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Foundations of Finance (B01.2311)

Numerical Example of the Arbitrage
When Calls Violate the Minimum Value Prior to Expiration

1. Prior to expiration, the minimum value is:

$$C \geq \text{Max}\ [0, S - Ee^{-rt}]$$

2. Suppose S = $101 E = $100 r = .06 t = 1 yr. Item (1) implies:

$$C \geq \text{Max}\ [0, \$101 - \$100\ (.9418)]$$
$$C \geq \$6.82$$

Note: $5.82 of the $6.82 minimum value comes from discounting E (= $100) at a continuously compounded 6% and $1.00 comes from the intrinsic value ($101-$100).

3. Assume C = 1, i.e., calls are selling at their intrinsic value and below their minimum value (they do not reflect the interest saved by delayed payment of E).

4. Set up the following arbitrage portfolio today:

	Cash Flow
Buy 1 call	-$1
Sell short 1 share	+$101
Invest $100 proceeds at .06	–

5. At the end of one year your portfolio looks like this:

Investment is worth $100 $e^{.06}$ =	$106.182
Long 1 call	
Short 1 share	

6. Evaluate what happens at the end if S > E and S ≤ E

<u>S > E</u>

	Cash Flow
Exercise long C	-$100
Deliver against short S	–
Receive proceeds of investment	<u>+$106.18</u>
Net	+$6.18

<u>S≤ E</u> (e.g., S = 98)

Leave call unexercised	$0
Buy S in market	-$98
Deliver S against short	–
Receive proceeds of investment	<u>+$106.18</u>
Net	+$8.18

7. Therefore:

(a) No matter what happens (S > E or S < E) the portfolio established "today" produces a riskless profit in one year.

(b) You will exploit this arbitrage as long as you can, driving up the value of C until, in this case, it is at least $6.82 (assuming the stock price stays at $101).

(c) The call value at the end reflects the interest rate (6%) that you save by not having to pay E (=$100) until the end. The arbitrageur, in fact, captures this interest when it is not embedded in the call. In particular, the minimum earned here is $6.18 which is equal to the amount of the mispricing $6.82 - $1.00 = $5.82, which becomes $6.18 (= $5.82 $e^{.06}$) at the end of one year.