

Simple Numerical Arbitrage Example
Demonstrating Put Call Parity (Assuming $r=0$ for simplicity)

1. Put call parity states

$$C = S - Ee^{-rt} + P$$

2. Assume $S = \$110$, $E = \$100$, $r = 0$, $t = \text{anything}$. Then Item (1) implies: $C = \$110 - \$100 + P$

Therefore,

If $P = \$2$, then $C = \$12$

If $P = \$5$, then $C = \$15$

3. Assume $S = \$110$, $E = \$100$, $r = 0$ and $t = 1$ period. Also assume $C = \$12$ and $P = \$5$

Thus the call is undervalued and/or the put is overvalued, given $S = \$110$ and $E = \$100$

4. Construct an arbitrage today by buying the "cheap" call at \$12 and selling the "expensive" put at \$5. Recall that buying C and selling P guarantees that you must buy S and pay out E . Therefore, we complete the arbitrage by selling S short at \$110 and investing the cash proceeds of the short sale, call purchase, and put sale at $r (= 0$ here but not in general).

Our position today is

	Cash Flow
Long 1 call	- \$ 12
Short 1 put	+ \$ 5
Short 1 share	+ \$110
Invest Proceeds in Treasuries	- <u>\$103</u>
Net	0

5. Examine what happens on expiration if $S > E$ and if $S < E$

	Cash Flow
<u>$S > E$</u>	
Receive Proceeds of Treasuries	+\$103
Exercise long C (get stock)	-\$100
Put expires worthless	0
Return short stock	<u>0</u>
Net	+\$3

	Cash Flow
<u>$S < E$</u>	
Receive Proceeds of Treasuries	+\$103
Call expires worthless	0
Put is exercised against you	-\$100
Take stock just received from put and return against short	<u>0</u>
	+\$3

6. Since the position shown in 4 is profitable and riskless, we do it as often as we can, driving up the value of C and driving down the value of P until they differ by exactly \$10. Thus C is driven up from \$12 and P is driven down from \$5. Whether C is \$14 and P is \$4 or C is \$15 and P is \$5 or whether C is \$12 and P is \$2 depends on the valuation of the call (or the put) from a model such as Black-Scholes. Put call parity just gives the relative value of C versus P given S and E.

7. Two Other Perspectives:

- a) The profit of \$3 is guaranteed no matter what happens because it equals the "mispricing" of $C - P$ versus $S - E$

$$C - P = \$12 - \$5 = \$7$$

$$S - E = \$110 - \$100 = \$10$$

- b) Another way of thinking about this is: Buying the call for \$12 and selling the put for \$5 allows you to be long the underlying stock (including exercise cost of \$100) for \$107. Since you sold the stock for \$110, you are entitled to \$3.